

## ALGEBRA AND DATA TYPES: CHEAT SHEET

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Type	Expr	Type	Expr	Type	Expr
!	0	(x, y)	$x * y$	String	String
()	1	Result<x, y>	$x + y$	Option<x>	$1 + x$
bool	2	[x; n]	$x^n$	HashSet<x>	$2^x$
u8	256	fn(x) -> y	$y^x$	Vec<x>	$\frac{1}{1-x}$

Algebraic Law	Equation	Description	Data Type Example
Commutivity	$x * y = y * x$	Reordering	$(x, y) \equiv (y, x)$
Associativity	$(x * y) * z = x * (y * z) = x * y * z$	Rearranging	$((x, y), z) \equiv (x, (y, z)) \equiv (x, y, z)$
Identity	$1 * x = x$	Unit Elimination	$(((), x) \equiv x$
Absorbtion	$0 * x = 0$	Never Contagion	$(!, x) \equiv !$
Commutivity	$x + y = y + x$	Reordering	Result<x, y> $\equiv$ Result<y, x>
Associativity	$(x + y) + z = x + (y + z) = x + y + z$	Rearranging	(can rearrange variants within enums)
Identity	$0 + x = x$	Never Elimination	Result<x, !> $\equiv$ x
Distributivity	$x * (y + z) = x * y + x * z$	Lowering Data	$(x, \text{Result}<y, z>) \equiv (\text{Result}<x, y>, \text{Result}<x, z>)$
Defn. of Exp.	$x^n = x * x * \dots * x$ ( <i>n</i> times)	Array $\leftrightarrow$ Tuple	$[x; 3] \equiv (x, x, x)$
Identity	$x^1 = x$	Short Array	$[x; 1] \equiv x$
Exp. Law 0	$1^x = 1$	Boring Array	$[(); 1] \equiv ()$
Exp. Law 1	$(x^n)^m = x^{(n*m)}$	Array Flattening	$[[x; n]; m] \equiv [x; n*m]$
Exp. Law 2	$x^{(n+m)} = x^n * x^m$	Array Splitting	$[x; n+m] \equiv ([x; n], [x; m])$
Exp. Law 3	$(x * y)^n = x^n * y^n$	AoS $\leftrightarrow$ SoA	$[(x, y); n] \equiv ([x; n], [y; n])$
Equiv. Reprs.	$x^n = x^n$	Func $\leftrightarrow$ Array	fn(u8) -> x $\equiv$ [x; 256]
Identity	$x^1 = x$	Short Func	fn<()> -> x $\equiv$ x
Exp. Law 0	$1^x = 1$	Boring Func	fn(x) -> () $\equiv$ ()
Exp. Law 1	$(z^y)^x = z^{(x*y)}$	Currying	fn(x) -> fn(y) -> z $\equiv$ fn(x, y) -> z
Exp. Law 2	$z^{(x+y)} = z^x + z^y$	Func Splitting	fn(Result<x, y>) -> z $\equiv$ (fn(x) -> z, fn(y) -> z)
Exp. Law 3	$(y * z)^x = z^x * y^x$	Func Splitting	fn(x) -> (y, z) $\equiv$ (fn(x) -> y, fn(x) -> z)